

Proof: worked examples

HSC Mathematics Extension 2

These examples show assessable skills, not an official HSC topic weighting or prediction. The 2024 NSW syllabuses have their first HSC examination in 2027; students sitting the 2026 HSC continue under the applicable 2017 syllabus.

Worked example 1 | 3 marks

Catalog reference: ext2_morris_2025_q16a_exponential_inequality | seed 41001 | mapping ext2-proof-1

An exponential inequality

Prove that if $x > 0$, then $e^{(3x)} > 1 + 3x$.

(a) Give a calculus proof. [3 marks]

Answer: Apply derivatives to $f(x) = e^{(3x)} - 1 - 3x$.

Auxiliary Function: Let $f(x) = e^{(3x)} - 1 - 3x$.

Derivative Sign: $f'(x) = 3(e^{(3x)} - 1) > 0$ for $x > 0$.

Inequality: $f(x) > f(0) = 0$.

Worked example 2 | 2 marks

Catalog reference: ext2_nor25_chained_triangle_inequality | seed 41001 | mapping ext2-proof-1

A chained absolute-value inequality

Prove $3(\text{abs}(a-b) + \text{abs}(c-b) + \text{abs}(c-d)) \geq 3(a-d)$.

(a) Give a direct proof. [2 marks]

Answer: Apply the triangle inequality to obtain $\text{abs}(a-c) + \text{abs}(c-d) \geq \text{abs}(a-d)$, then use $\text{abs}(a-d) \geq a-d$.

Triangle Inequality: $\text{abs}(a-b) + \text{abs}(b-c) \geq \text{abs}(a-c)$

Absolute Value Bound: $\text{abs}(a-d) \geq a-d$

Worked example 3 | 3 marks

Catalog reference: ext2_ind25_reciprocal_log_inequality | seed 41001 | mapping ext2-proof-1

A logarithmic inequality

For $a > 1$ and $b > 1$, prove $\log_a(a^{b^3}) + \log_b(a^{(3)} b) \geq 8$.

(a) Give a complete proof. [3 marks]

Answer: Set $t = \log_a(b) > 0$. The left side is $2 + 3(t + 1/t)$, and $(t - 1/t)^2 \geq 0$ gives $t + 1/t \geq 2$.

Change Of Base: $\log_b(a) = 1/\log_a(b)$.

Nonnegative Square: $t + 1/t \geq 2$.

Substitute Bound: 8

Source and methodology

Examples are deterministic curriq catalog generations from existing QDSL archetypes at the published fixed seed. The generator copies only student-facing prompts, mark allocations, topic mappings, display tables or options, checked answers and marking-path results into this resource.

Official syllabus source:

<https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-extension-2-11-12-2024/overview/course>

Teacher verification note: check the mathematics, notation, permitted technology and syllabus fit for your cohort before classroom or assessment use.