

Proof: worked examples

HSC Mathematics Extension 1

These examples show assessable skills, not an official HSC topic weighting or prediction. The 2024 NSW syllabuses have their first HSC examination in 2027; students sitting the 2026 HSC continue under the applicable 2017 syllabus.

Worked example 1 | 3 marks

Catalog reference: ext1_sgs25_induction_odd_square | seed 41001 | mapping ext1-p1-1

Induction and divisibility

Prove by induction that $9^{(2n)} - 1$ is divisible by 8 for all integers $n \geq 1$.

(a) Give a complete induction proof. [3 marks]

Answer: Verify $n=1$, assume the result for $n=k$, then write the next expression as 81 times the hypothesis plus $81-1$, both divisible by 8.

Induction Base: 80

Induction Step: 81

Worked example 2 | 3 marks

Catalog reference: ext1_jr25_induction_power_divisibility | seed 41001 | mapping ext1-p1-1

Induction and divisibility

Prove by induction that $15^{(2^n)} - 1$ is divisible by $2^{(n+2)}$ for every positive integer n .

(a) Give a complete induction proof. [3 marks]

Answer: The base case follows because $15^{(2^1)} - 1$ is divisible by 8. Assuming $15^{(2^k)} - 1 = 2^{(k+2)}r$, factor the next expression as the product of this term and $15^{(2^k)} + 1$, whose second factor is even.

Induction Base Case: 8 divides $15^{(2^1)} - 1$.

Induction Factorisation: $A^{(2^2)} - 1 = (A - 1)(A + 1)$.

Induction Conclusion: The product contains $2^{(k+3)}$.

Worked example 3 | 3 marks

Catalog reference: ext1_pem_2025_q12b_induction_divisibility | seed 41001 | mapping ext1-p1-1

Induction and divisibility

Use mathematical induction to prove that $3^{(2n)} + 7$ is divisible by 8 for every positive integer $n \geq 1$.

(a) Write a complete induction proof. [3 marks]

Answer: Verify the base case, assume $3^{(2k)} + 7 = 8m$, and rewrite the next term as $8(9m - 71)$.

Induction Base Case: $3^{(2^1)} + 7 = 16$ is divisible by 8.

Induction Hypothesis: Assume $3^{(2k)} + 7 = 8m$.

Induction Step: Substitute into $3^{(2(k+1))} + 7$ and factor out 8.

Source and methodology

Examples are deterministic curriq catalog generations from existing QDSL archetypes at the published fixed seed. The generator copies only student-facing prompts, mark allocations, topic mappings, display tables or options, checked answers and marking-path results into this resource.

Official syllabus source:

<https://curriculum.nsw.edu.au/learning-areas/mathematics/mathematics-extension-1-11-12-2024/overview/course>

Teacher verification note: check the mathematics, notation, permitted technology and syllabus fit for your cohort before classroom or assessment use.